Scaling theory of localisation

Consider the conductance G of a system.

In a system of very large size L (hypercube) it is related to conductivity as

$$G = \sigma L^{d-2}$$

However, conductivity σ is a good characteristic of the system only when $L \gg \xi$, where the correlation length ξ diverges at the transition

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<u>Scaling hypothesis</u>: $\beta(G) = \dfrac{d \ln G}{d \ln L}$ is a universal function depends only on G and the dimension, but is independent of microscopic details

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When G is large $\beta \to d-2$

(when $G \to \infty$)

When G is very small

$$G \approx G_0 e^{-\frac{L}{\xi}} \to \ln G = \ln G_0 - \frac{L}{\xi}$$

$$\to \beta = -\frac{1}{\xi}\frac{dL}{d\ln L} = -\frac{L}{\xi} = \ln\frac{G}{G_0}$$

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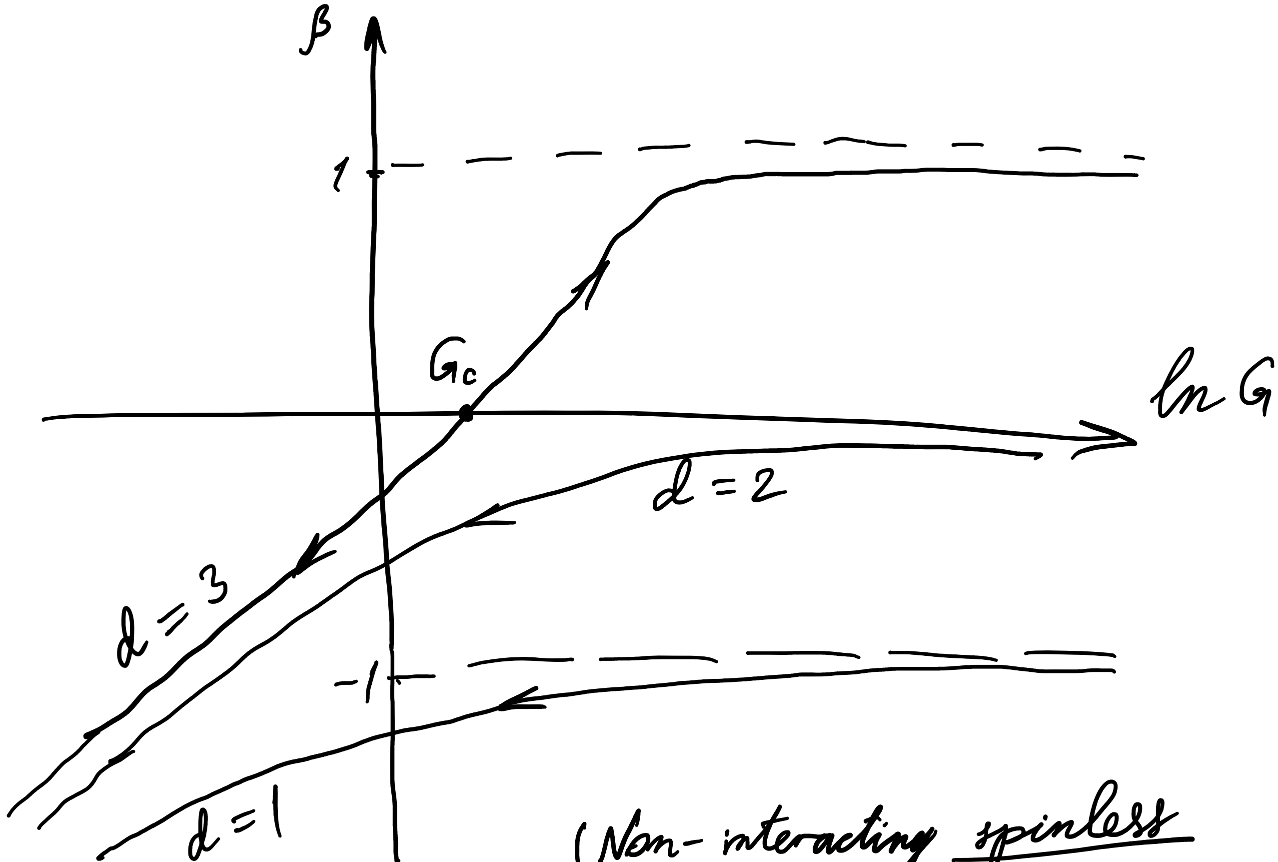


(Non-interacting spinless electrons)

$\beta > 0$ - metal:
take a cube of size L, make a larger sample of such cubes → conductance increases

$\beta < 0$ - insulator:
when adding cubes, conductance decreases

Consider in more detail the 2D case. The scaling theory implies everything is

The scaling theory implies everything is localised in 2D.

We know the Drude conductivity in 2D

$$\sigma = e^2 D \nu_F \,, \quad \nu_F = \frac{m}{2\pi} = \frac{k_F}{2\pi v_F}$$

$$D = \frac{v_F^2 \tau}{2}$$

$$\rightarrow \sigma = \frac{e^2}{4\pi} k_F l$$

Recovering $\hbar$, $\sigma_{Drude} = \frac{e^2}{4\pi\hbar} k_F l$

With the WL correction,

$$\boxed{\begin{aligned} \sigma &= \frac{e^2}{4\pi\hbar} k_F l - \frac{e^2}{\pi^2\hbar} \ln \frac{L}{l} \\ &= \sigma_0 - \frac{e^2}{\pi^2\hbar} \ln \frac{L}{l} \end{aligned}}$$

The WL is cut off by the sample size (in the absence of phonons or other dephasing mechanisms)

The localisation length:

$$\xi \simeq l \, e^{\pi^2 \sigma_0 / (e^2/\hbar)}$$

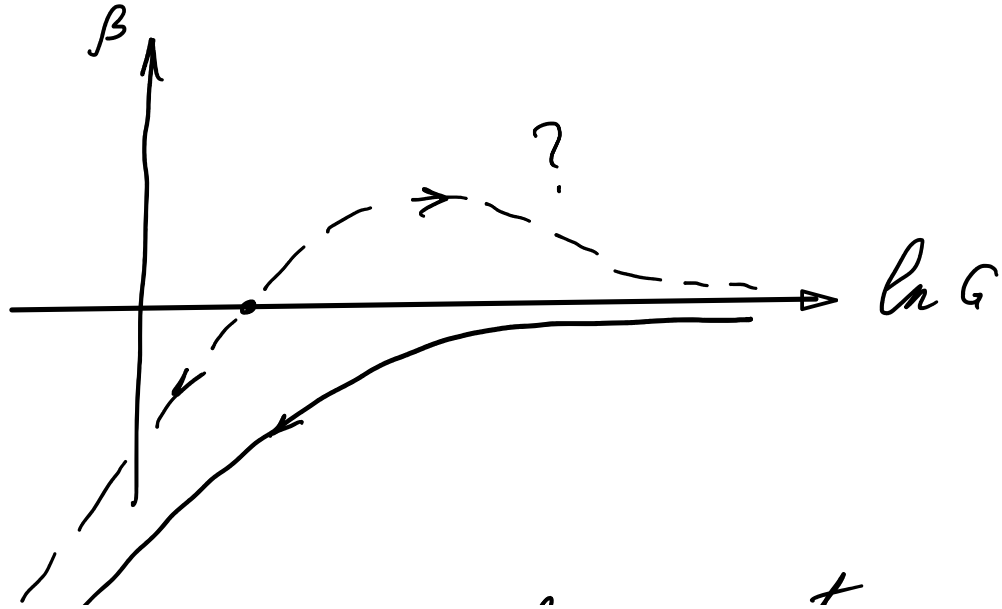
$$\boxed{\xi \simeq \ell \; e}$$

Exponentially sensitive to the conductivity (= Ioffe-Regel parameter)

Consistent with the scaling theory of localisation. The localisation length is exponentially sensitive to the amount of disorder in the sample !!!

If it's a film of finite thickness b, this adds an extra factor $(k_F b)$ to the exponent

However...



Possible in systems with spin-orbit interactions!!

$$\delta\sigma \sim -\frac{e^2}{\hbar}\int_{\tau}^{\tau_\varphi}\frac{dt}{t}\left(\frac{3}{2}e^{-\frac{t}{\tau_{so}}}-\frac{1}{2}\right)$$

While $\tau_\varphi < \tau_{so}$, the WL will be negative, then it starts to grow

$$\tau \ll \tau_{so} \ll \tau e^{c \cdot k_F l}$$

when $k_F l \sim 1$ it's impossible to satisfy

when $k_F l \gg 1$ dephasing kicks in